Topic: Intermediate value theorem

Question: Assume the value of $f(a)$ is less than the value of $f(b)$. Use the definition of the Intermediate Value Theorem to write an inequality that represents the root of the function on the given interval.

$f(x)$ on the interval $[a, b]$

Answer choices:

A $f(a) \leq 0 \leq f(b)$

B $f(a) > 0 > f(b)$

C $f(a) \geq 0 \geq f(b)$

D $f(a) < 0 < f(b)$

© Krista King Math

Solution: D

The Intermediate Value Theorem states that, for any function $f(x)$ that's continuous on an interval $[a, b]$, if that function has a root in the interval (crosses the x-axis), then we can define the relationship between the endpoints as

$$f(a) < 0 < f(b)$$

or

$$f(a) > 0 > f(b)$$

Since we were told that $f(a)$ is less than $f(b)$, we know the inequality that represent's the root is $f(a) < 0 < f(b)$.

© Krista King Math

Topic: Intermediate value theorem

Question: Is the Intermediate Value Theorem valid for both open and closed intervals?

Answer choices:

A It's valid for closed intervals only.

B It's valid for open intervals only.

C It's valid for both open and closed intervals.

D An interval isn't necessary for the Intermediate Value Theorem.

© Krista King Math

Solution: A

The Intermediate Value Theorem requires that the function be continuous on the entire interval in order to be valid. This means that the Intermediate Value Theorem only works for closed intervals.

© Krista King Math

Topic: Intermediate value theorem

Question: Use the Intermediate Value Theorem to find the root of the function on the given interval.

$$x^2 + 2x - 35 = 0$$

on the interval $[0,10]$

Answer choices:

A $x = -7$

B $x = 5$

C $x = -7, 5$

D No real root exists in the interval

© Krista King Math

Solution: B

The root of a function is the point at which that function is equal to zero, which is the point where the function crosses the x-axis. To find the roots of the given function, we'll factor the left-hand side.

$$x^2 + 2x - 35 = 0$$

$$(x + 7)(x - 5) = 0$$

$$x + 7 = 0$$

$$x = -7$$

or

$$x - 5 = 0$$

$$x = 5$$

We've found two roots of the function, but only $x = 5$ is in the interval $[0,10]$, so it's the only root we need to consider.

We should also double-check the endpoints of the interval to confirm that the value of the function at the left endpoint is negative and that the value of the function at the right endpoint is positive. If we can verify $f(a) < 0 < f(b)$, then by the Intermediate Value Theorem we'll prove that the function must have a root in the interval, and we already know that the root is $x = 5$.

Plugging in the endpoints of the interval, we get

$$f(0) = (0)^2 + 2(0) - 35$$

© Krista King Math

$$f(0) = -35$$

and

$$f(10) = (10)^2 + 2(10) - 35$$

$$f(10) = 85$$

Since we know that $f(a) < 0 < f(b)$, or in our case $-35 < 0 < 85$, there must be a root in the interval, and the root is $x = 5$.

© Krista King Math